Section 8.1 (page 203)

8.1 1. Affirming the consequent is invalid.
     4. Weakening the Antecedent invalid.
     7. Constructive Dilemma is valid.

8.4 Here is an informal proof of the argument:

     The unicorn, if horned, is elusive and dangerous.
     If elusive or mythical, the unicorn is rare.
     If a mammal, the unicorn is not rare.
     The unicorn, if horned, is not a mammal.

     Proof. Assume that the unicorn is horned. We want to prove that it is not a mammal. Assume, by way of a proof by contradiction, that it is a mammal. By the third premise, the unicorn is not rare. By the first premise, the unicorn is elusive. But then by the second premise, the unicorn is rare, after all. This gives us our contradiction, showing us that our assumption is false.

8.7 Here is an informal proof of the argument:

     $b$ is small unless it’s a cube.
     If $c$ is small, then either $d$ or $e$ is too.
     If $d$ is small, then $c$ is not.
     If $b$ is a cube, then $e$ is not small.
     If $c$ is small, then so is $b$.

     Proof. Assume that $c$ is small. We want to prove that $b$ is small. Assume, by way of contradiction, that $b$ is not small. Then by the first premise, $b$ is a cube. Then by the final
premise, \( \varepsilon \) is not small. But then by the second premise, \( \delta \) is small. But the third premise assures us that if \( \delta \) is small, then \( \zeta \) isn’t small, contradicting our initial assumption. Hence, using proof by contradiction and conditional proof, we see that if \( \zeta \) is small, then so is \( \eta \).

Section 8.2 (page 212)

8.18 One counterexample to Affirming the Consequent can be found in the world and sentence file below. You should come up with a different counterexample.
8.21 The same world used in 8.18 provides a counterexample to the argument shown below. You should find a different counterexample.

8.24

<table>
<thead>
<tr>
<th></th>
<th>1. A ∨ B</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.</td>
<td>A → C</td>
</tr>
<tr>
<td>3.</td>
<td>B → D</td>
</tr>
<tr>
<td>4.</td>
<td>☑ ☑ → Elim: 2,4</td>
</tr>
<tr>
<td>5.</td>
<td>C</td>
</tr>
<tr>
<td>6.</td>
<td>☑ ☑ v Intro: 5</td>
</tr>
<tr>
<td>7.</td>
<td>☑</td>
</tr>
<tr>
<td>8.</td>
<td>☑ ☑ → Elim: 3,7</td>
</tr>
<tr>
<td>9.</td>
<td>C ∨ D</td>
</tr>
<tr>
<td>10.</td>
<td>☑ ☑ v Intro: 8</td>
</tr>
<tr>
<td></td>
<td>C ∨ D</td>
</tr>
<tr>
<td></td>
<td>☑ ☑ v Elim: 7-9,4-6,1</td>
</tr>
</tbody>
</table>

8.29 Hint: A partial proof of 8.29 is shown below. We have used Taut Con to justify one instance of Excluded Middle. If your instructor does not allow this, then he or she is mean and you will need to replace this step by a proof of P ∨ ¬P.
8.32 Hint: A partial formalization of the informal proof we gave of 8.4 above is shown below.

1. Horned(c) → (Elusive(c) ∧ Dangerous(c))
2. (Elusive(c) v Mythical(c)) → Rare(c)
3. Mammal(c) → ∼Rare(c)

4. ▼ Horned(c)
5. ▼ Mammal(c)
6. ∼Rare(c)
7. ▼
8. ⊥
9. ∼Mammal(c)
10. Horned(c) → ∼Mammal(c)
8.35 Hint: A partial formalization of the informal proof we gave of 8.7 above is shown below.

```
1. \neg\text{Cube}(b) \to \text{Small}(b)
2. \text{Small}(c) \to (\text{Small}(d) \lor \text{Small}(e))
3. \text{Small}(d) \to \neg\text{Small}(c)
4. \text{Cube}(b) \to \neg\text{Small}(e)
5. \bowtie \text{Small}(c)
6. \bowtie \neg\text{Small}(b)
7. \bowtie \neg\text{Cube}(b)
8. \text{Small}(b)
9. \bot
10. \text{Cube}(b)
11. \neg\text{Small}(e)
12. \text{Small}(b)
13. \text{Small}(b)\to \text{Small}(b)
```

8.39 Part of the truth table for 8.39 is shown below, enough of it to show that there is a row where the premises are true and the conclusion false in that row. (We cut off the rest just to save space here.) Hence the conclusion is not a tautological consequence of the premises. And hence, by Soundness, the conclusion cannot be proved from the premises in the system $\mathcal{F}_T$. 

Section 8.3 (page 221)
### Section 8.4 (page 223)

**8.44** Hint: This is invalid, so you will need to construct a counterexample.

**8.45** Hint: This is valid, so you will need to construct a proof. It will need to use \(\neg\ \text{Intro}\) and \(\ = \ \text{Elim}\).
8.46 Hint:

1. Cube(a) v (Cube(b) → Tet(c))
2. Tet(c) → Small(c)
3. (Cube(b) → Small(c)) → Small(b)
4. ~Cube(a)
5. Cube(a)
6. Rule?:
7. Cube(b) → Small(c)
8. Cube(b) → Tet(c)
9. Rule?:
10. Cube(b) → Small(c)
11. Cube(b) → Small(c) ✓ ¥ vElim: 8-10,5-7,1
12. Small(b) ✓ ¥ → Elim: 11,3

8.50 Hint: Remember that if Cube(a) and Cube(c) are both true or both false, then Cube(a) ↔ Cube(c) is true.

8.53

1. Small(a) → Small(b)
2. Small(b) → (SameSize(b, c) → Small(c))
3. ~Small(a) → (Large(a) ∧ Large(c))
4. SameSize(b, c)
5. Small(a) v ~Small(a) ✓ ¥ Taut Con:
6. Small(a)
7. (Large(c) v Small(c)) ✓ ¥ Taut Con: 6,1,2,4
8. ~Small(a)
9. (Large(c) v Small(c)) ✓ ¥ Taut Con: 8,3
10. (Large(c) v Small(c)) ✓ ¥ vElim: 8-9,6-7,5
11. SameSize(b, c) → (Large(c) v Small(c)) ✓ ¥ → Intro: 4-10